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PHASE SYNCHRONIZATION BETWEEN TWO DIFFERENT OSCILLATORS WITH UNIDIRECTIONAL SIGNAL COUPLING

J. Y. CHEN and K. W. WONG
Department of Computer Engineering and Information Technology, City University of Hong Kong, Hong Kong, China

J. W. SHUAI
Department of Physics and Astronomy, Ohio University, Athens, OH 45701, USA

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A control scheme is applied between two different oscillators to study their phase synchronization. It utilizes unidirectional signal coupling and only measures the time interval when the trajectories to the two oscillators' attractors cross the Poincaré surfaces respectively. By using this scheme, phase synchronization (without 2π phase slips) can be obtained between two different chaotic systems whose signal variables have large amplitude mismatch. This unidirectional signal coupling also provides a minimum information flow from the driving system to the response system. Therefore it can be used in synchronizing systems with substantially different dynamics via a channel with low information rate.

Keywords: Chaos; phase synchronization; Poincaré surface; chaotic oscillators.

The study of synchronization is a fundamental research interest with applications in a variety of fields [Pikovsky *et al.*, 2001; Boccaletti *et al.*, 2002]. In general, this phenomenon is usually obtained through proper control schemes that utilize coupling among elements. Various control schemes are investigated in order to apply the synchronization in engineering fields [Pecora & Carroll, 1990; Dai & Ma, 2001; Murakami & Ohtsubo, 2001; Xiao *et al.*, 1996; Chen, *et al.*, 1999]. Moreover, several types of synchronization have been investigated extensively, including complete synchronization [Xiao *et al.*, 1996; Chen *et al.*, 1999], generalized synchronization [Abarbanel *et al.*, 1996; Lewis *et al.*, 2001; Zheng *et al.*, 2000], and phase synchronization (PS) [Chen *et al.*, 2001, 2002a, 2002b; Tang & Heckenberg *et al.*, 1997; Liu *et al.*, 2001; Shuai & Durand, 1999; Maza *et al.*, 2000; Parlitz *et al.*, 1996; Schäfer *et al.*, 1998]. In particular, PS corresponds to an entrainment of the phases of chaotic oscillators, whereas their amplitudes remain chaotic and noncorrelated. This phenomenon is also closely related to phase-locked loop that is highly relevant to engineering applications [Roland, 1997]. To investigate PS, a well-defined phase variable $\phi_{1,2}$ has to be measured in both coupled systems. In classical definition, PS occurs if the difference $|\phi_1 - \phi_2|$ between the corresponding phases is bounded within a chosen value. A weaker synchronization, referred to as imperfect PS [Rosenblum *et al.*, 1996, 1997; Zaks *et al.*, 2000], is the coincidence of the average frequencies defined as $\Omega_1 = \Omega_2$ with $\Omega_{1,2} = \lim_{t\to\infty} \phi_{1,2}(t)/t$ while their phase differences are unbounded.

The results of PS studies may be applied in technical and experimental fields where a coherent superposition of several chaotic output channels is desired. PS has been found useful in natural systems such as extended ecological system [Blasius *et al.*, 1999, 2000], magnetoencephalographic activity of Parkinsonian patients [Tass *et al.*, 1998], electrosensitive cells of the paddlefish [Neiman *et al.*, 1999], and solar activity [Palus *et al.*, 2000], as well as laboratory experiments such as circuits [Parlitz *et al.*, 1996], lasers [Allaria *et al.*, 2001; Lariontsev, 2000; Kozyreff *et al.*, 2000], and plasmas [Ticos *et al.*, 2000]. In laser systems, PS among lasing elements can be achieved using proper control scheme of coupling in an array of semiconductor lasers as it is important to obtain a large output power concentrated in a single-lobed far field pattern [Kozyreff *et al.*, 2000].

In order to uncover the properties of PS extensively, numerous direct signal couplings, such as one way coupling [Pazó *et al.*, 2000] and asymmetric coupling [Zheng *et al.*, 2000], have been studied. However, most of these couplings can only be used in two slightly nonidentical systems. If they are adopted in two different systems where interactive dynamics are substantially different, only imperfect PS can be found. By this means, it is necessary to find an effective control scheme of coupling that can achieve PS in this case.

Besides direct signal coupling, bidirectional signal coupling has also been investigated in two chaotic high-dimensional systems whose dynamics are substantially different [Boccaletti *et al.*, 2000]. With this type of coupling, PS can be obtained even with a small noise perturbation. The result has potential application in the control of chaos. On the other hand, due to the weak synchronization nature of PS, it is easy to be achieved by partial signals. An example is the binary coupling that takes only two possible values $\{-1, 1\}$ and provides a minimal information flow from the drive subsystem to the response one [Parlitz & Wedekind, 2000]. However, only imperfect PS is obtained between two different oscillators using the binary coupling.

In this article, we propose a unidirectional signal coupling method to synchronize two different systems. Similar to binary coupling, it also uses relative strong coupling, but with only a small amount of information to be transmitted. Moreover, PS (without 2π phase slips) can be obtained between two different chaotic systems with substantial difference in their amplitudes. The coupling term appears in intermittent coupling when the coupling strength exceeds the PS transition value. We will also give a detailed analysis on the mechanism that results in PS between different coupled systems.

We study the unidirectional coupling with the interaction of the hyperchaotic and the chaotic Rössler oscillators [Rössler, 1979]. The drive–response system is governed by the following equations.

$$\begin{aligned}
\dot{x}_1 &= -y_1 - z_1\,,\\
\dot{y}_1 &= x_1 + 0.25y_1 + w\,,\\
\dot{z}_1 &= 3.0 + x_1 z_1\,,\\
\dot{w} &= -0.5z_1 + 0.05w\,,\\
\dot{x}_2 &= -\omega y_2 - z_2 + C\,,\\
\dot{y}_2 &= \omega x_2 + 0.15y_2\,,\\
\dot{z}_2 &= 0.2 + z_2(x_2 - 10)\,.
\end{aligned} \tag{1}$$

where the parameter ω is the natural frequency of the Rössler oscillator. As the mean frequency $\Omega_2 \approx \omega$ for the Rössler oscillator, various values of Ω_2 can be obtained by changing ω. A typical and extensively-discussed scheme of direct signal coupling can be expressed as:

$$C(t) = \varepsilon(x_1(t) - x_2(t)) \tag{2}$$

where ε is the coupling constant. If the trajectory has a rotation center, we can choose a Poincaré section in a proper way. As a result, the number of times q crosses the Poincaré section can be obtained. Thus, we can develop a unidirectional signal coupling method:

$$C(t) = \varepsilon[x_2(t) - x_2(t-\tau)]\tanh[q_1 - q_2] \tag{3}$$

with τ as a small time delay. In this example, we select $y_i = 0$ $(i = 1, 2)$ as the Poincaré surfaces. They are shown in Figs. 1(a) and 1(b). Only when the trajectories cross the Poincaré surface would q_i be incremented by one. The function tanh() is used to obtain the boundary values of C.

With the coupling term stated in Eq. (3), only the number of crossing times in the Poincaré section, but not the amplitude, is used in the coupling. By this means, their amplitudes can maintain noncorrelation with each other at various strengths of coupling. This control method is especially useful for the coupling of two different systems whose signal variables have large amplitude mismatch. For the system described by Eq. (1), the maximum amplitude of x corresponding to the hyperchaotic and the chaotic Rössler attractors are about 130 and 25, respectively. Although the size of the hyperchaotic Rössler attractor is about five times larger than that of the chaotic Rössler, the unidirectional coupling scheme can still yield PS.

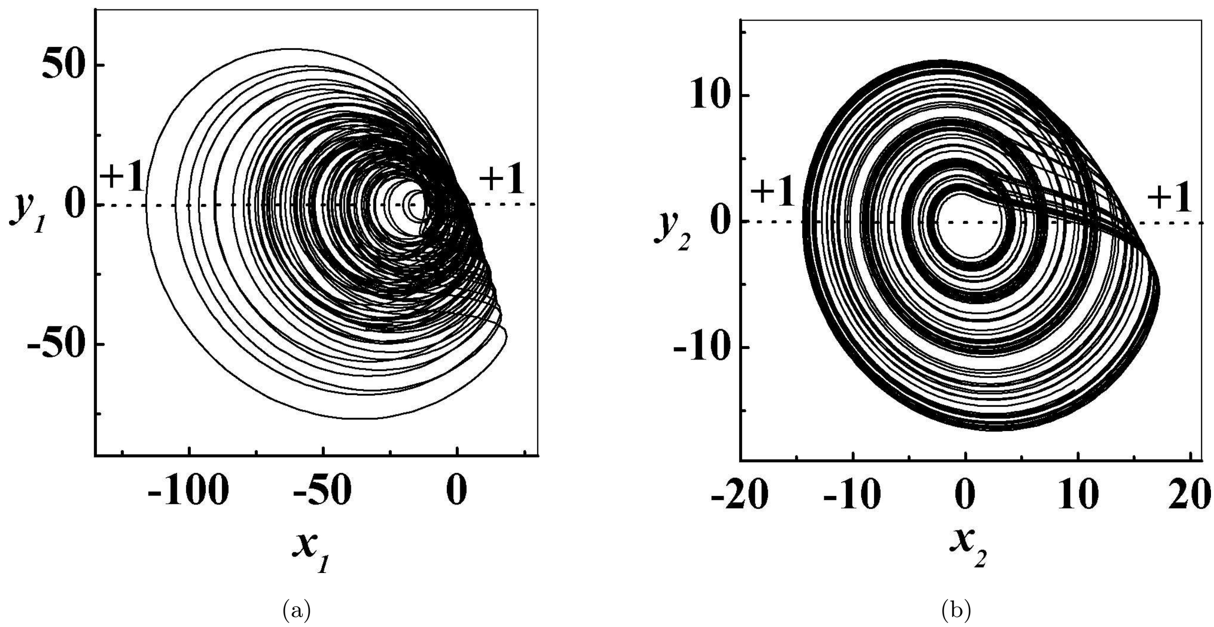


Fig. 1. The phase portrait of (a) hyperchaotic and (b) chaotic Rössler oscillators on the x–y plane. Here, $\omega = 1$ and $C = 0$. Throughout this article, Eq. (1) is numerically solved using the fourth-order Runge–Kutta method. The initial conditions of the hyperchaotic Rössler oscillator are set to $x_1(0) = -20.0$, $y_1(0) = 0.0$, $z_1(0) = 0.0$, and $w(0) = 15.1$.

To observe PS we must define a suitable phase for the oscillators. Since the phase portraits of the hyperchaotic and the chaotic Rössler oscillators in the $x - y$ plane explicitly show a rotational trajectory around the center (x_0, y_0), as found in Fig. 1, the phases can be conveniently defined as

$$\phi_i = \tan^{-1}\left[\frac{y_i - y_{i(0)}}{x_i - x_{i(0)}}\right] \quad i = 1, 2 \tag{4}$$

where the center of hyperchaotic Rössler oscillator is $(x_{1(0)}, y_{1(0)}) = (-15, 0)$ and that of chaotic Rössler oscillator is $(x_{2(0)}, y_{2(0)}) = (0, 0)$. Although the rotation center of the hyperchaotic Rössler attractor is small, the trajectory runs strictly around the small rotation center and no funnel shape is found [Kim *et al.*, 2000]. Figure 1 shows the case when ϕ_i increases by π as y_i crosses zero. This corresponds to the increment of q_i by one. By this means, q_i has the same trend of increment as ϕ_i, i.e. $q_i \propto \phi_i$ except that the increment of q_i is discrete and fixed at each step. This measurement is similar to the phase measured from Poincaré points [Pikovsky *et al.*, 1997; Hu & Zhou, 2000]. In the latter case, the phase increases by 2π each time when the trajectory passes through the Poincaré point. However, the former method requires more selected positions to sense the frequency of the trajectory. It is not necessary to obtain the exact phase value when the trajectory passes the selected values, only the ratio of increment is required.

Simulation results corresponding to the two coupling methods governed by Eqs. (2) and (3) are shown in Fig. 2. Figures 2(a) and 2(c) show the results of Eq. (1) with direct signal coupling [i.e. Eq. (2)]. In Fig. 2(a), the difference between two mean frequencies, i.e. $\Delta\Omega = \Omega_1 - \Omega_2$, is found with small agitation. When $\varepsilon = 0.005$ and 0.01, there is no strict flat region at $\Delta\Omega = 0$, as marked by a dotted line. The two $\Delta\Omega$ curves are either above or below zero. The clear agitation at some particular points on the dotted line $\Delta\Omega = 0$ indicates that the PS is not stable. The phase difference $\Delta\phi$ $(= \phi_1 - \phi_2)$ is plotted in Fig. 2(c) whose situation corresponds to $\Delta\Omega \approx 0$. It has intermittent 2π phase slips with a long phase locking duration between two adjoining slips. Evidently, Figs. 2(a) and 2(c) show that Eq. (1) with direct signal coupling can only lead to imperfect PS. If ε increases, the single center of Rössler oscillator is destroyed and the oscillator becomes funnel in shape. This is due to the fact that the two different oscillators in Eq. (1) have large amplitude mismatch from each other. However, with the coupling defined in Eq. (3), PS is found. The results are shown in Figs. 2(b) and 2(d). In Fig. 2(b), it is clear that a large flat region of coupling is obtained for $\Delta\Omega = 0$. This phenomenon

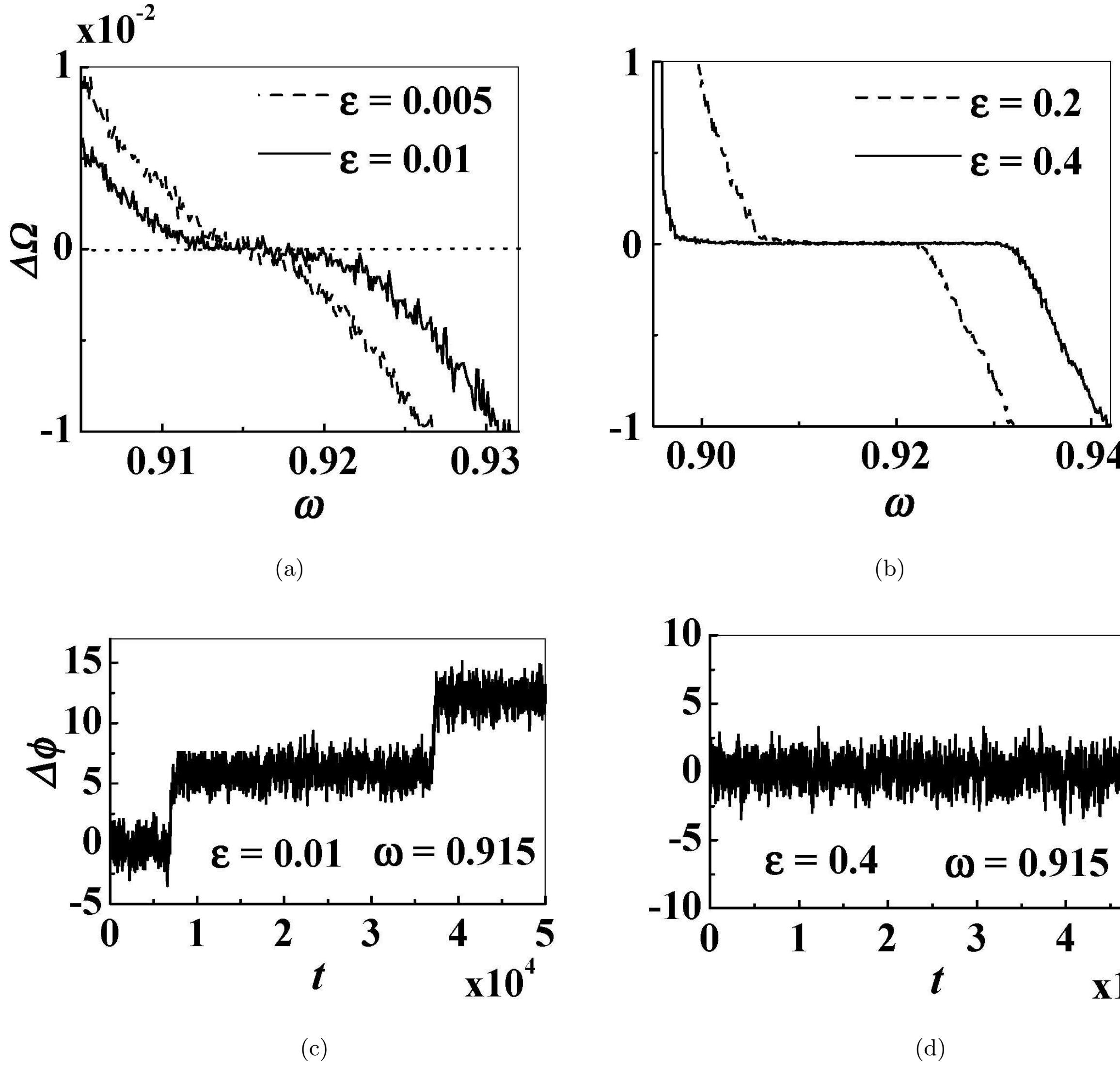
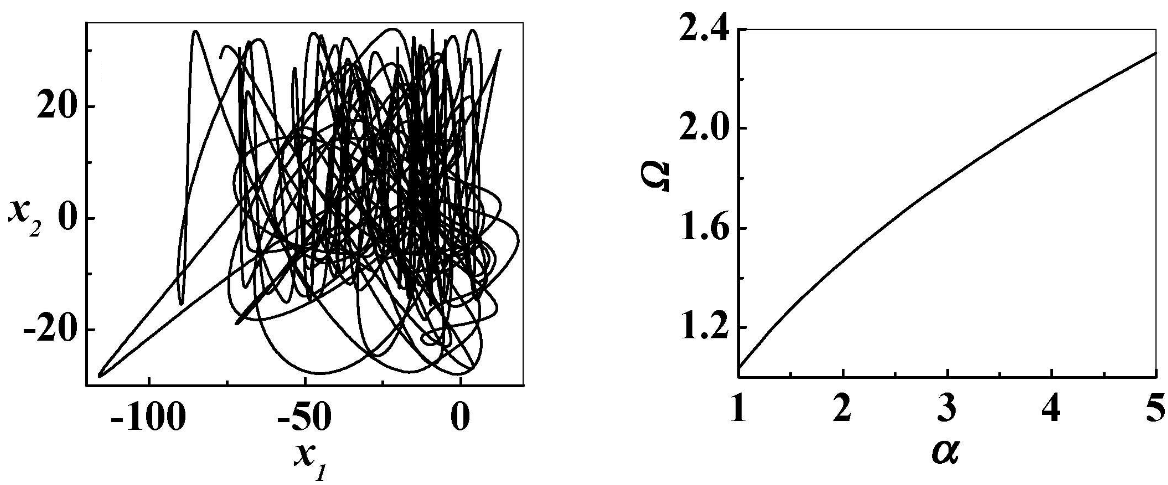


Fig. 2. Mean frequency difference $\Delta\Omega$ and phase difference $\Delta\phi$ for the system described by Eq.(1) using two coupling schemes governed by Eqs. (2) and (3), respectively. $\Delta\Omega$ versus parameter ω at several values of the coupling strength ε using (a) the signal coupling scheme in Eq. (2), and (b) the unidirectional signal coupling scheme in Eq. (3). The time series of $\Delta\phi$ in Eq. (1) under the condition (c) $\varepsilon = 0.01$, $\omega = 0.915$ using the signal coupling, and (d) $\varepsilon = 0.4$, $\omega = 0.915$ using the unidirectional signal coupling. The time delay is $\tau = 0.1$ for (b) and (d).

indicates that the PS is stable. Traditionally, the PS is bounded with $|\Delta\phi| < \pi$, as observed from Fig. 2(d). In this case, their amplitudes are still non-correlated even for a relatively strong coupling. This is clearly shown in Fig. 3(a).

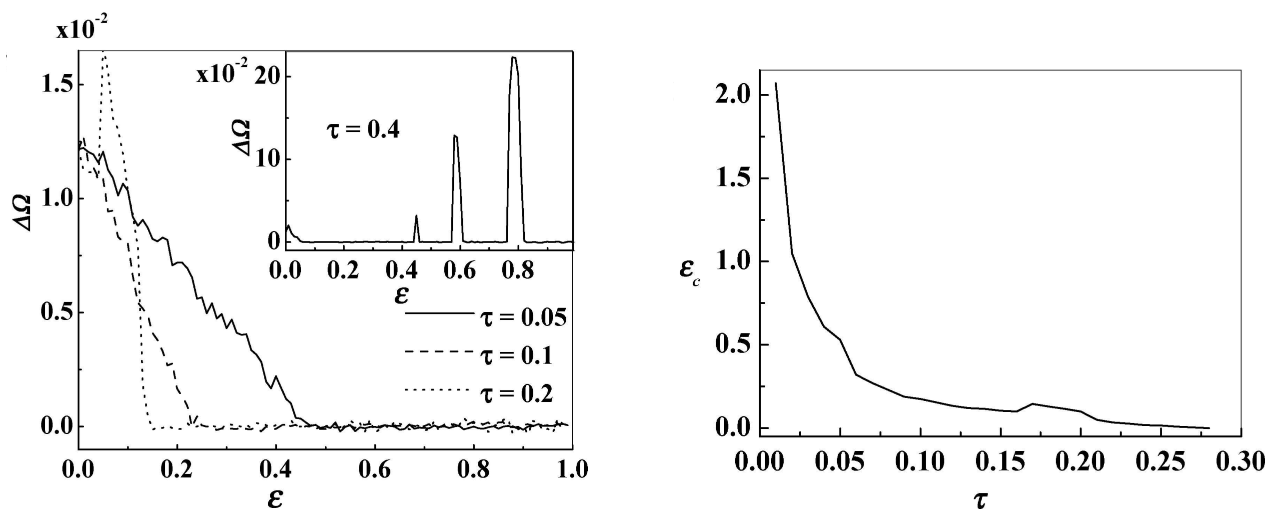
For the unidirectional signal coupling, feasible values of time delay τ have to be chosen. Simulation results with various values of τ are shown in Fig. 4(a). When $\tau < 0.2$, PS can always be found, but at different phase transitions. An increase in τ leads to a reduction in the phase transition. A portrait between the coupling strength ε_c at PS transition and the time delay is shown in Fig. 4(b). With an increase of time delay, ε_c decreases dramatically. However, too large the value of τ may reduce the robustness of PS. For example, when $\tau = 0.4$, the PS is corrupted occasionally. This is shown in the inner plot of Fig. 4(a). This implies that in order to obtain stable PS, there are no strict limits on the selection of τ provided that it is small enough to obtain robust PS.

It is important to uncover the mechanism of unidirectional signal coupling with which PS can be obtained between two different oscillators. For



(a) (b)

Fig. 3. (a) Projections of the attractor of the coupled oscillators described by Eq. (1) on the (x_1, x_2) plane at $\varepsilon = 0.4$ and $\tau = 0.1$. (b) The mean frequency Ω in Eq. (6) versus the parameter α.



(a) (b)

Fig. 4. (a) The portraits of mean frequency difference $\Delta\Omega$ versus coupling strength ε at various time delays τ. The inner plot shows the unstable PS with a relatively large τ. These simulations are performed with the parameter $\omega = 0.905$ in Eq. (1). (b) The PS transition at various time delays.

$\tau \ll 1$, Eq. (3) can be rewritten in the following approximate form:

$$C(t) = \varepsilon \tanh[q_1 - q_2](\tau \dot{x}_2 + \xi(t)) \tag{5}$$

where ξ is a small noise perturbation obtained by

$$\xi(t) = x_2(t) - x_2(t-\tau) - \tau\dot{x}_2$$

$$= \frac{\tau^2}{2}\ddot{x}_2 + O(\tau^3) \tag{6}$$

Here, $O(\tau^3)$ represents the third and higher order terms of the Taylor series expansion on $x_2(t-\tau)$. Substituting it into Eq. (1), the first formula of the Rössler oscillators then becomes

$$\dot{x}_2 = \alpha(t)(-\omega y_2 - z_2 + \delta(t)) \tag{7}$$

where

$$\alpha(t) = (1 - \varepsilon \tanh[q_1 - q_2])^{-1} \tag{8}$$

$$\delta(t) = \xi\varepsilon \tanh[q_1 - q_2] \tag{9}$$

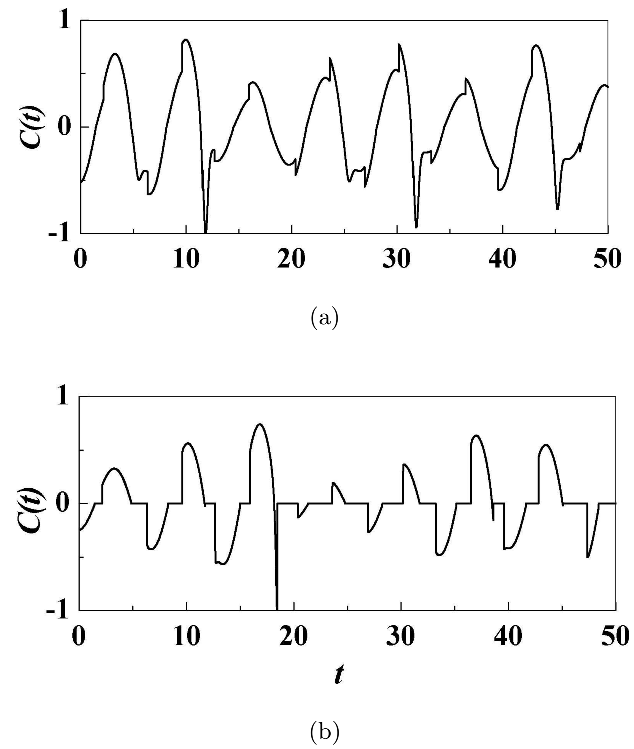


Fig. 5. Time evolution of the coupling term $C(t)$ in both non-PS and PS states. (a) Non-PS state at $\tau = 0.1$ and $\varepsilon = 0.2$. (b) PS state at $\tau = 0.1$ and $\varepsilon = 0.5$.

When $\tau \ll 1$, the perturbation ξ shown in Eq. (6) is so small that it can be ignored. Then we obtain

$$\dot{x}_2 = \alpha(t)(-\omega y_2 - z_2) \tag{10}$$

If α is increased from 1 to 5 in Eq. (10), Ω increases almost linearly with α, i.e. $\alpha \propto \Omega$. This is shown in Fig. 3(b). Furthermore, from Eq. (8), it is evident that $\varepsilon \tanh(q_1 - q_2) \propto \alpha$. Therefore, $\varepsilon < \tanh(q_1 - q_2) > \propto \Omega$ where $\langle \bullet \rangle$ represents mean value. This direct proportional relationship leads to PS in Eq. (1) and can be considered as a type of phase locked loop control [Roland, 1997]. For example, if $\Omega_1 > \Omega_2$ at a particular time, we will find that $q_1 > q_2$ after some iterations. From Fig. 3(b), it is found that Ω_2 will increase in the subsequent iterations. By this means, when ε is large enough, the mean frequencies are basically equal to each other. On the other hand, if we select a relatively large τ, the noise perturbation ξ increases. Moreover, with the increase of ε, the overall perturbation $\varepsilon \tanh[q_1 - q_2]\xi$ is enlarged significantly. Thus, the above explanation for Fig. 3(b) will be influenced by the large overall perturbation and PS is corrupted occasionally at relative large ε, as observed in the inner plot of Fig. 4(a).

The coupling term $C(t)$ has different characteristics at non-PS and PS states. In non-PS state, the phase difference increases with time and makes $q_1 - q_2$ distant from zero. After a certain time interval, $|q_1 - q_2|$ is so large that $\tanh(\cdot) \to 1$ or -1. The time evolution of $C(t)$ is mainly determined by ε and $(x_2(t) - x_2(t - \tau))$, as observed from Eq. (3). The result is shown in Fig. 5(a). However, in PS state, their phase difference agitates around zero. At some duration, $q_1 - q_2 = 0$ and it indicates zero coupling, i.e. $C(t) = 0$ at certain instants. By this means, the coupling term in PS state corresponds to intermittent coupling, as observed in Fig. 5(b). It is mainly produced by the discrete measurement of q_i. Suppose that at a certain time, PS is obtained with $q_1 - q_2 = 0$ and $\Delta\phi \approx 0$. For subsequent time, $|\Delta\phi|$ starts to increase as $q_1 - q_2 = 0$. However, since q_i is measured only at the Poincaré surfaces, there is a small time duration before $|\Delta\phi|$ is reflected by a nonzero value of $|q_1 - q_2|$. In other words, in the small time duration, $|\Delta\phi|$ increases, but $|q_1 - q_2|$ is kept zero. As a result, the intermittent coupling phenomenon is found and $|\Delta\phi|$ fluctuates around zero.

From the above discussion, it is evident that the control scheme of unidirectional signal coupling only utilizes the number of times the trajectory passes through the Poincaré surface. The drive system transmits "1" to the response one once the trajectory passes through the Poincaré surface. At the response system, the parameter q_1 performs $q_1 + 1$ at the response site after receiving the binary signal "1". By this means, lesser signal information is required from the drive to the response systems than from direct signal coupling.

In conclusion, based on the dynamical properties of signal coupling, we have developed a control scheme of unidirectional signal coupling that results in PS between different oscillators, while their amplitudes are maintained noncorrelated. After PS, the intermittent coupling phenomenon is observed. The dynamics of unidirectional signal coupling is analyzed in detail. This control scheme may have potential in applications that require PS between two different oscillators where their amplitudes have large mismatch, such as the output summation of different generators with coherence in phase. It may also be used for synchronizing analog circuits via a digital channel with a low information rate.

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